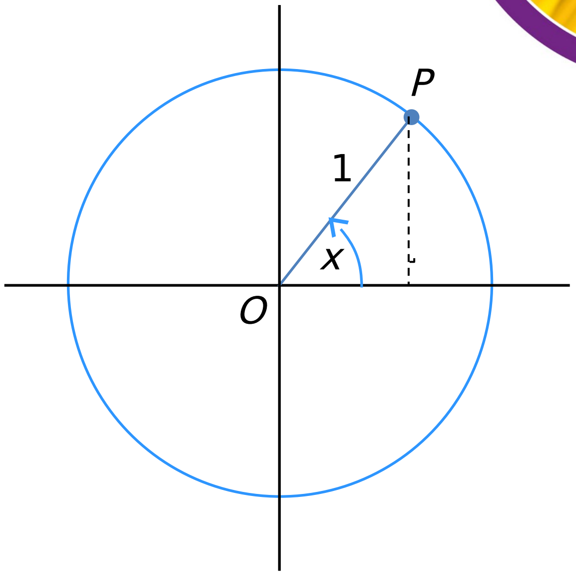
TRIGOMOMETRIC EQUATIONS AND GRAPHS

The graphs of y = sin x and y = cos x



In the diagram the line *OP* is of length 1 unit and *OP* makes an angle x with the positive horizontal axis.

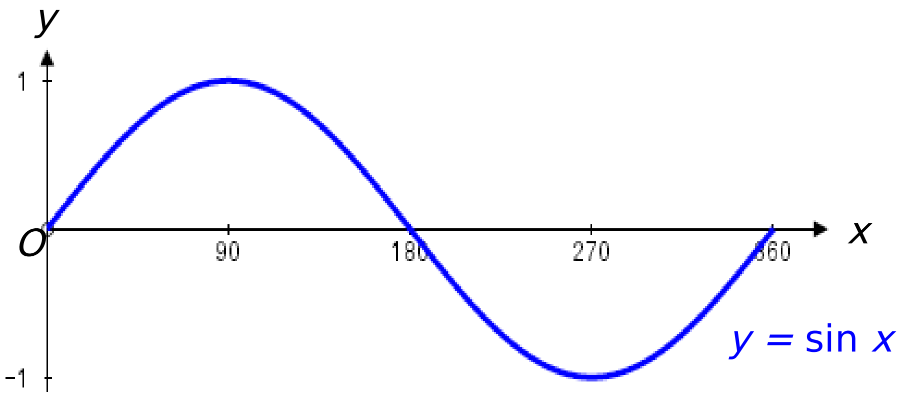
Using trigonometry:

- height of right-angled triangle = sin x
- base of right-angled triangle = cos x

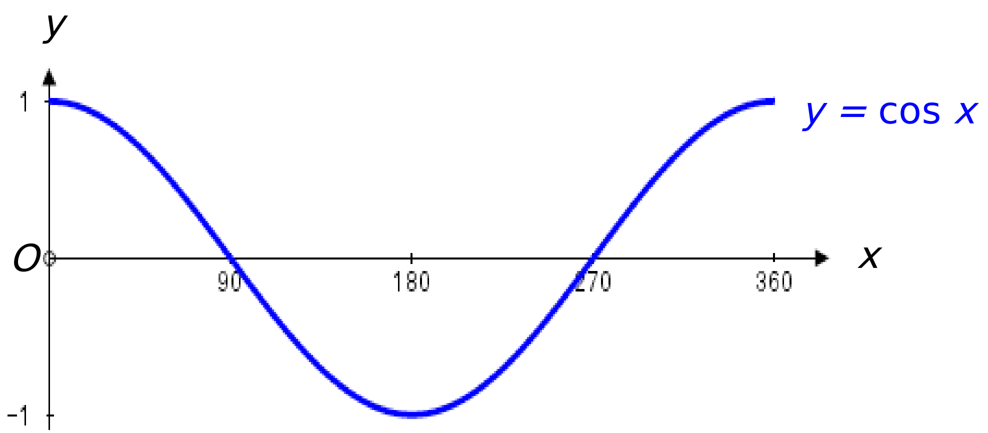
The height of *P* above the horizontal axis changes from $0 \rightarrow 1 \rightarrow 0 \rightarrow -1 \rightarrow 0$.

The displacement of *P* from the vertical axis changes from $1 \rightarrow 0 \rightarrow -1 \rightarrow 0 \rightarrow 1$.

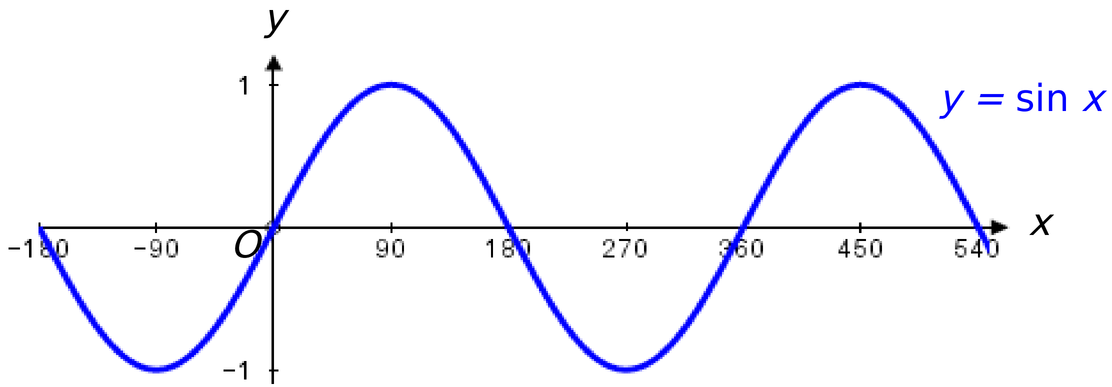
The graph of $y = \sin x$ for $0° \leq x \leq 360°$ is:

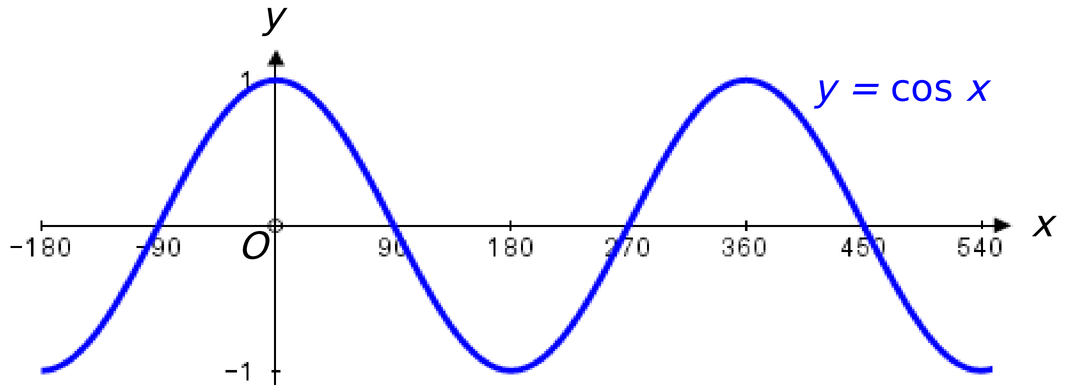


The graph of $y = \cos x$ for $0° \leq x \leq 360°$ is:



The graphs of $y = \sin x$ and $y = \cos x$ can be expanded beyond $0° \leq x \leq 360°$:





The sine and cosine functions are called **periodic functions** because they repeat themselves over and over again.

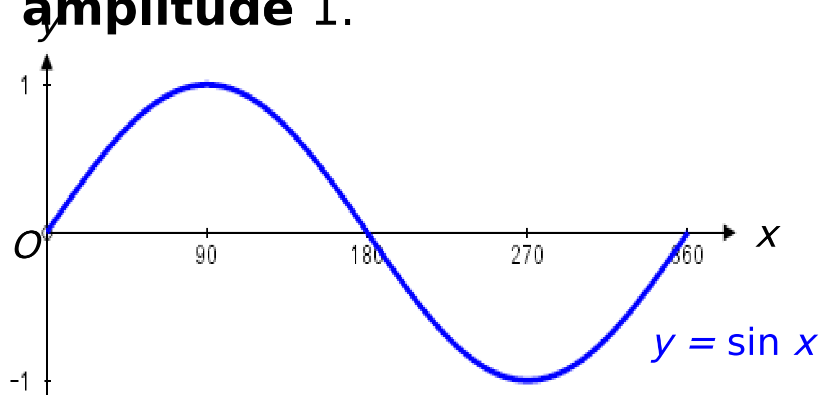
The **period** of a periodic function is defined as the length of one repetition or cy

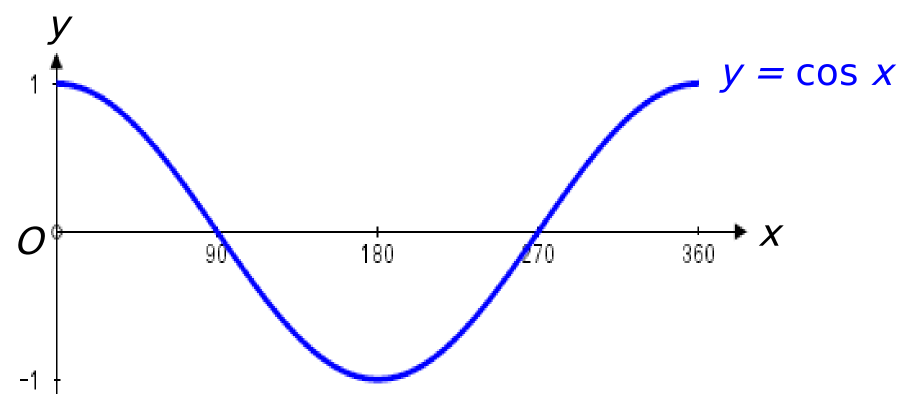
The basic sine and cosine functions repeat every 360°.

We say they have a **period** of 360°.

The **amplitude** of a periodic function is defined as the distance between a maximum (or minimum) point and the principal axis.

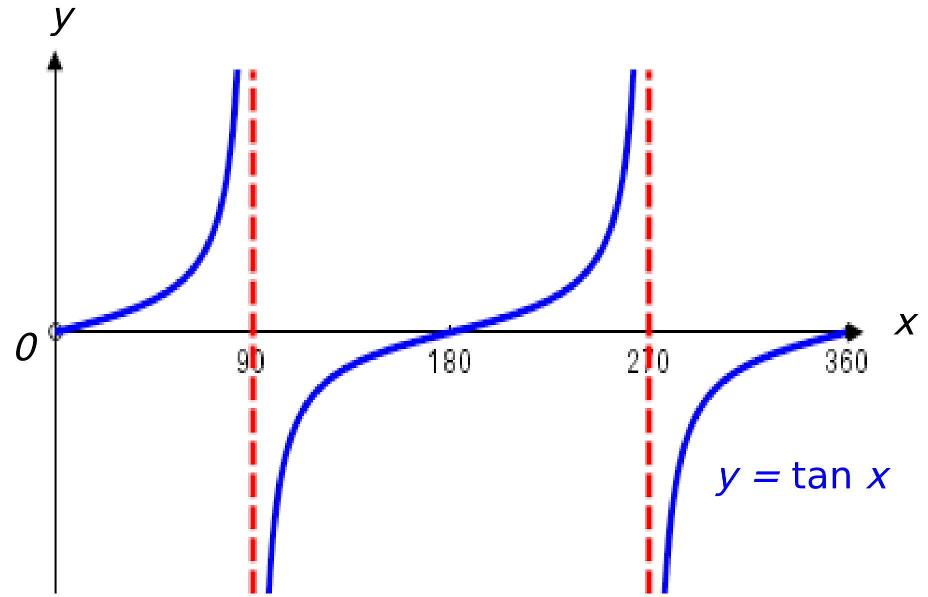
The basic sine and cosine functions have **amplitude** 1.





The graph of $y = \tan x$

The tangent function behaves very differently to the sine and cosine functions.



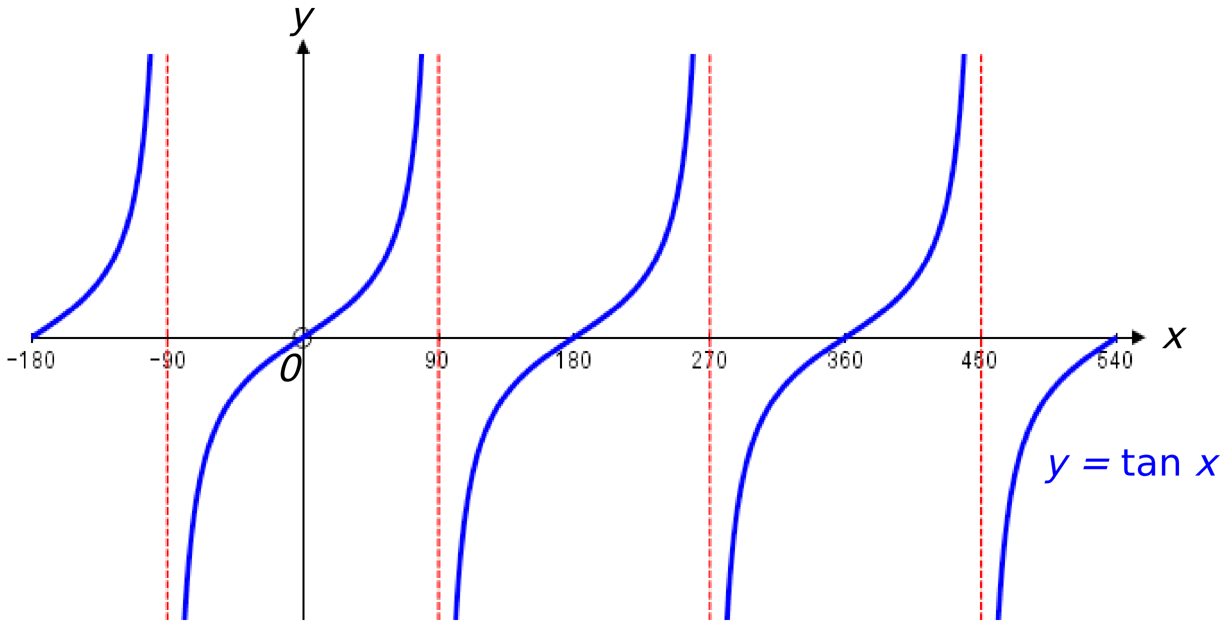
The red dashed lines at $x = 90°$ and $x = 270°$ are **asymptotes**.

The branches of the graph get closer and closer to the asymptotes without ever reaching them.

The tangent function repeats its cycle every 180° so its period is 180°.

The tangent function does not have an amplitude.

The graph of $y = \tan x$ can be expanded beyond $0° \leq x \leq 360°$:



Solving trigonometric equations for values between 0° and 360°

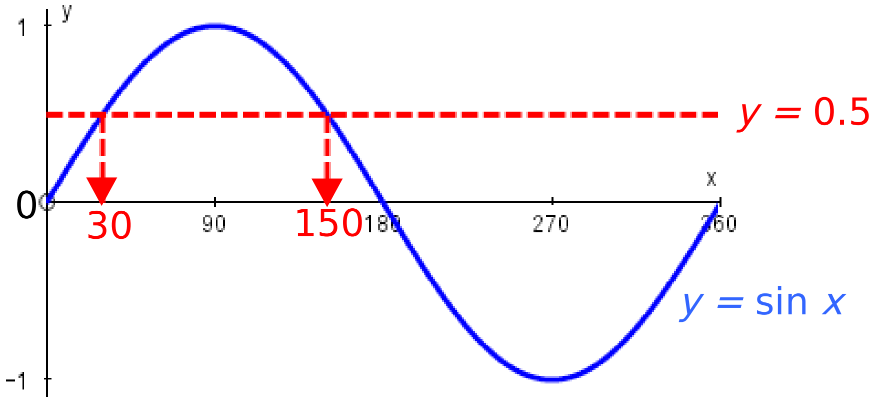
Consider solving the equation: $\sin x = 0.5$ for $0° \leq x \leq 360°$.

$$x = \sin^{-1}(0.5)$$

A calculator gives the answer: $x = 30°$

There is, however, a second value of x for which $\sin x = 0.5$

This can be found by considering the symmetry of the curve $y = \sin x$:



The second value $= 180° - 30° = 150°$

Hence the solution of the equation $\sin x = 0.5$ for $0° \leq x \leq 360°$ is

$$x = 30° \text{ or } 150°$$

Example

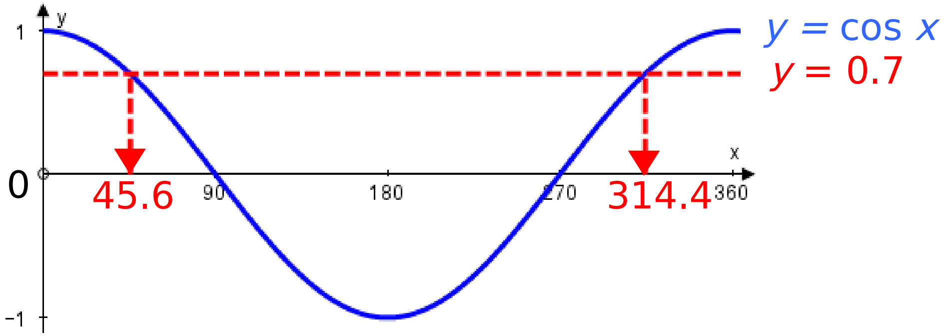
1 Solve $\cos x = 0.7$ for $0° \leq x \leq 360°$.

$\cos x = 0.7$

$x = \cos^{-1}(0.7)$

$x = 45.6°$

A sketch graph of $y = \cos x$ is used to find any other values:



The second value $= 360° - 45.6° = 314.4°$

Hence the solution of the equation $\cos x = 0.7$ for $0° \leq x \leq 360°$ is

$x = 45.6°$ or $314.4°$

Example

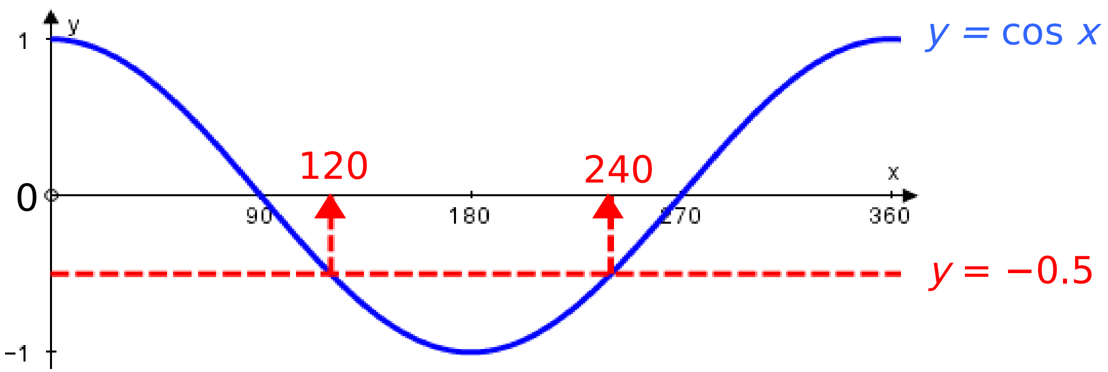
2 Solve $\cos x = -0.5$ for $0° \leq x \leq 360°$.

$\cos x = -0.5$

$x = \cos^{-1}(-0.5)$

$x = 120°$

A sketch graph of $y = \cos x$ is used to find any other values:



The second value $= 360° - 120° = 240°$

Hence the solution of the equation $\cos x = -0.5$ for $0° \leq x \leq 360°$ is

$x = 120°$ or $240°$

Example

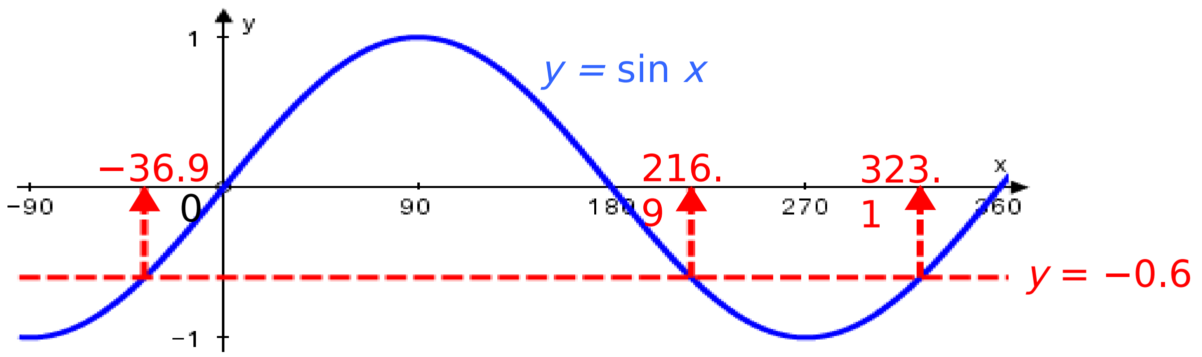
3 Solve $\sin x = -0.6$ for $0° \le x \le 360°$.

$\sin x = -0.6$

$x = \sin^{-1}(-0.6)$

$x = -36.9°$ (this angle is out of range)

A sketch graph of $y = \sin x$ is used to find any other values:



$x = 180° + 36.9° = 216.9°$ or $x = 360° - 36.9° = 323.1°$

Hence the solution of the equation $\sin x = -0.6$ for $0° \le x \le 360°$ is

$x = 216.9°$ or $323.1°$

Example

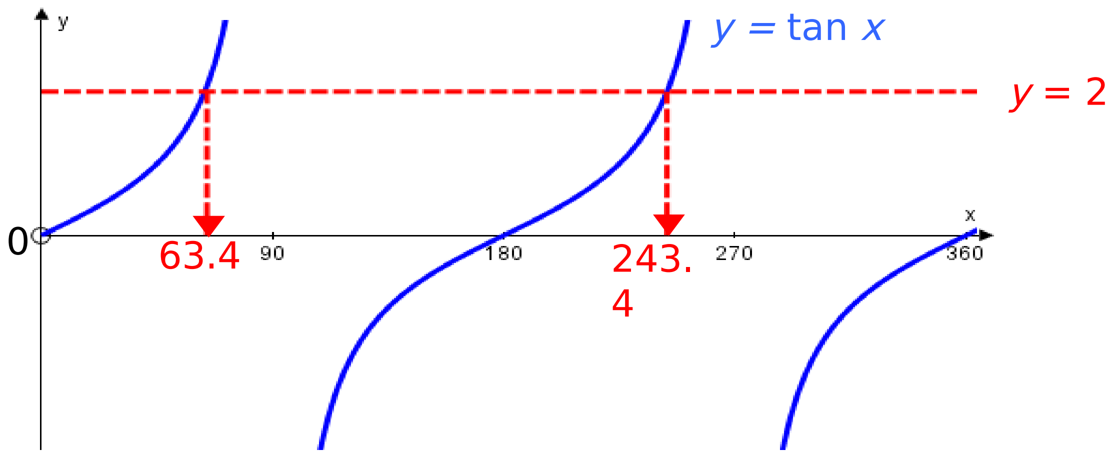
4 Solve $\tan x = 2$ for $0° \le x \le 360°$.

$\tan x = 2$

$x = \tan^{-1}(2)$

$x = 63.4°$

A sketch graph of $y = \tan x$ is used to find any other values:



The second value $= 180° + 63.4° = 243.4°$

Hence the solution of the equation $\tan x = 2$ for $0° \le x \le 360°$ is

$x = 63.4°$ or 243.41